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A p-adic cohomological approach to congruences of meromorphic modular forms

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Numerical observation [Pengcheng Zhang '25]

$$C : y^2 + xy = x^3 - x^2 - 2x - 1, \quad j(C) = -3375$$

$$\boxed{a_{np^{\ell+1}}\left(\frac{E_4}{j+3375}\right) \equiv \mu^2 \cdot a_{np^{\ell}}\left(\frac{E_4}{j+3375}\right) \pmod{p^{3\ell}}}$$

for $p > 3$ ordinary prime for C and μ unit root of $X^2 - a_p(C)X + p$.

U_p**-action** on meromorphic modular forms $\longleftrightarrow$ **crystalline data** of C.

X modular curve, $\mathcal{E}$ universal elliptic curve, $\alpha \in X$, $(\mathcal{H}_2, \nabla)$ vector bundle with connection

$$\begin{array}{ccccccccc} 0 & \longrightarrow & H^1_{\mathrm{dR}}(X, \mathcal{H}_2) & \longrightarrow & H^1_{\mathrm{dR}}(X \setminus \{\alpha\}, \mathcal{H}_2) & \xrightarrow{\mathrm{Res}} & \mathrm{Sym}^2 H^1_{\mathrm{dR}}(\mathcal{E}_\alpha)[1] & \longrightarrow & 0 \\ & & & & \downarrow \wr & & & & \\ & & & & M_4^{\mathrm{mero},\alpha} / \theta^3 M_{-2}^{\mathrm{mero},\alpha} & & & & \end{array}$$

These meromorphic modular forms represent **Frobenius equivariant splitting** of filtered-φ exact sequence.

The cohomological machine

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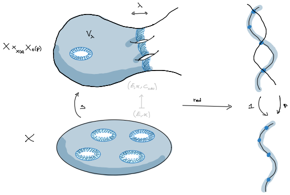
- **Canonical subgroup** construction $\Rightarrow$
  Frobenius neighborhood and relation to U_p**-operator**
- **Crystalline cohomology** $\Rightarrow$ integral
  Frobenius stable lattice to deduce congruences

Explicit modular action of Frobenius on cohomology classes.



Further directions (joint with H. Hassan)

- $X = \Gamma\backslash\mathcal{H}_p$ Shimura curve admitting **p-adic uniformization**;
- Study exact sequences of filtered (φ, N)-modules arising from extension by CM points;
- Compute Nekovár **p-adic height pairing** of such extensions;
- Provide p-adic analytic formula for **higher p-adic Green's functions** $\mathcal{G}_k$,
- We expect the value of $\mathcal{G}_k$ at CM point to be the p-adic $\log$ of an algebraic number.