GIVEN $\omega \in H^0(C, \Omega^1)$ WE HAVE $(\omega|_U, \omega|_V, 0) \in Z^1$

FOR WHICH INJECTIVITY FOLLOWS FROM GLUING $x_U|_{U\cap V} - x_V|_{U\cap V} = 0$.

BY CECH COMPLEX

$$H^1(C, \mathcal{O}_C) = \mathcal{O}_C(U\cap V) / \{f|_{U\cap V} - g|_{U\cap V} \mid f \in \mathcal{O}_C(U),\ g \in \mathcal{O}_C(V)\}$$

TAKE MAP

$$Z^1 \longrightarrow H^1(C, \mathcal{O}_C)$$
$$(\omega_U, \omega_V, f) \longmapsto [f]$$

WHERE KERNEL IS EXACTLY B^1.

ASSUME $g \geq 1$, LET $D = K$ CANONICAL DIVISOR

$$\ell(K + nP) - \ell(nP) = \cancel{\deg}\ 2g - 2 + n + 1 - g = n + g - 1$$

$$\ell(K + nP) > 0$$

LET $n = -\operatorname{ord}_Q df$, $\exists\, g \in K(C)$ $\omega_U = df + g\omega \in \Omega(U)$

$\omega_V = g\omega \in \Omega(V)$.

CRISTALLINE COHOMOLOGY I

§ INTRODUCTION AND MOTIVATION

- RECAP ON WEIL COHOMOLOGIES:

  X SMOOTH PROPER OVER $\mathbb{Z}_p$, FIX $i : \mathbb{Q}_p \hookrightarrow \mathbb{C}$

$$\begin{array}{ccccc} X_{\mathbb{C}} & \longrightarrow & X & \longleftarrow & X_{\mathbb{F}_p} \\ \downarrow & & \downarrow & & \downarrow \\ \operatorname{Spec}\mathbb{C} & \longrightarrow & \operatorname{Spec}\mathbb{Z}_p & \longleftarrow & \operatorname{Spec}\mathbb{F}_p \end{array}$$

  TO $X_{\mathbb{C}}(\mathbb{C})$ ASSOCIATE COMPLEX MANIFOLD X^{an}

  SO FAR WE HAVE SEEN
  - $H_{sing}(X^{an}, \mathbb{Z})$
  - $H_{dR}(X_\square)$ $\square \in \{\emptyset, \mathbb{C}, \mathbb{F}_p\}$, $H_{dR}(X^{an})$
  - $H_{\acute{e}t}(X_\square, \mathbb{Z}_\ell) = \varprojlim H_{\acute{e}t}(X_\square, \mathbb{Z}/\ell^i\mathbb{Z}_X)$ $\ell \neq p$

- ℓ-ADIC COHOMOLOGY IS ENDOWED W/ Frob_p ON $X_{\mathbb{F}_p}$ GIVING ZETA FUNCTION

$$P_i = \det(1 - \mathrm{Frob}_p t \mid H^i_c(X \otimes \overline{\mathbb{F}}_p, \mathbb{Q}_\ell))$$

REM $\ell \neq p$ CONDITION GIVES PROPER BASE CHANGE THEOREM

$\ell = p$ FAILS (IN SPECTACULAR WAY)

CONSIDER ARTIN - SCHREIER EXACT SEQUENCE FOR ÉTALE TOPOLOGY

$$0 \to \underline{\mathbb{Z}/p\mathbb{Z}}_{X_{\mathbb{F}_p}} \longrightarrow \mathcal{O}_{X_{\mathbb{F}_p}} \xrightarrow[1-\mathrm{Frob}_p]{} \mathcal{O}_{X_{\mathbb{F}_p}} \longrightarrow 0$$

W/ $\mathrm{Frob}_p : X_{\mathbb{F}_p} \longrightarrow X_{\mathbb{F}_p}$, $a \mapsto a^p$ ABSOLUTE FROBENIUS MORPHISM

RECALL $H^i_{ét}(X_{\mathbb{F}_p}, \mathcal{O}_X) = H^i_{ZAR}(X_{\mathbb{F}_p}, \mathcal{O}_X)$ FOR EVERY $i \geq 0$ IN GENERAL FOR $\mathcal{F}$ QUASI-COH SHEAF AND X QUASI-SEP, QUASI-COMPACT

THIS IMPLIES $H^i(X_{\mathbb{F}_p}, \underline{\mathbb{Z}/p\mathbb{Z}}_X) = 0 \quad \forall i > d.$

$\rightsquigarrow H^i(X_{\mathbb{F}_p}, \mathbb{Z}_p) = 0 \quad \forall i > d.$

• DE RHAM COHOMOLOGIES

⌜GROTHENDIECK'S IDEA (1968): DEFINE P-ADIC COHOMOLOGY OF X IN TERMS OF DE RHAM COHOMOLOGY OF LIFTS IN char 0 ⌟

CONSIDER k PERFECT FIELD OF char p , $p > 0$

$W(k) = W$ RING OF WITT VECTORS

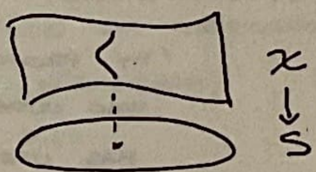
LET X SMOOTH PROPER OVER k

LET $f : \mathcal{X} \longrightarrow \mathrm{Spf}\, W[[t_1, \dots, t_r]] = S$ UNIVERSAL DEFORMATION OF X,

I.E. FOR EVERY SCHEME Z/W SMOOTH AND PROPER LIFTING X, $Z \times \mathrm{Spec}\, k = X$

THERE IS A POINT $x \in S(W)$ ST

$$Z \simeq \mathcal{X} \times_S W = \mathcal{X} \otimes_{W[[t]]} (W, x)$$



LET $\mathcal{H}^i = R^i f_* \Omega^\bullet_{\mathcal{X}/S}$ RELATIVE DE RHAM COHOMOLOGY

SUPPOSE $\mathcal{H}^i$ FREE OF FINITE RANK COMPATIBLE W/ BASE CHANGE

$$H^i_{dR}(Z, W) = x^* \mathcal{H}^i = \mathcal{H}^i \otimes_{W[[t]]} (W, x)$$

$\mathcal{H}^i$ IS ENDOWED W/ NATURAL CONNECTION

$$\nabla_{GM} : \mathcal{H}^i \longrightarrow \mathcal{H}^i \otimes \Omega^1_{S/W} \qquad \text{GAUSS-MANIN CONNECTION}$$

IF Z_1, Z_2 ARE TWO LIFTS OF X/k CORR TO $x, y : W[[t]] \longrightarrow W$

SUCH THAT $x \equiv y \bmod p$ THEN WE HAVE AN ISOMORPHISM

$$\chi(x,y) : \mathcal{H}^i \otimes (W, x) \longrightarrow \mathcal{H}^i \otimes (W, y)$$

$$\chi(x,y)(s) = \sum_{|n| \geq 0} \frac{(x(t) - y(t))^n}{n!} \left(\nabla_{GM} \left(\frac{\partial}{\partial t} \right)^n s \right)(y)$$

W/ COMPATIBILITIES $\chi(x,y)\,\chi(y,z) = \chi(x,z)$, $\chi(x,x) = \mathrm{id}.$

EX PROVE THE ISOMORPHISM ASSUMING ∇ NILPOTENT

HINT: FIX BASIS $e_1, \dots, e_r$ OF $\mathcal{H}^i$

$$\nabla(f e_i) = df \cdot e_i + f \sum \theta_{ij} dt_j$$

$$\nabla = d + \sum M_j \, dt_j \qquad M_j \in \mathbb{E}\mathrm{nd}(\mathcal{H}^i) = M_r(W[[t]])$$

PARALLEL TRANSPORT: FIND SECTION m OF $\mathcal{H}^i$ ST

- $\nabla(m) = 0$
- $x^* m = s$

REM ALGEBRAIC VS ANALYTIC DE RHAM

FOR X SMOOTH PROPER $/\mathbb{C}$ $\rightsquigarrow X^{an}$ COMPLEX ANALYTIC VARIETY

$\underline{\mathbb{C}}_X \longrightarrow \Omega^\bullet_{X^{an}}$ QUASI-ISOMORPHISM BY HOLOMORPHIC POINCARÉ LEMMA

C^∞ VERSION, ACCORDING TO DE RHAM, WAS PROVED BY VOLTERRA 1889 BUT ATTRIBUTED TO POINCARÉ

INDUCING ISOMORPHISM $H^\bullet(X^{an}, \underline{\mathbb{C}}_X) \simeq H^\bullet_{dR}(X^{an}) = \mathbb{H}^\bullet(X^{an}, \Omega^\bullet_{X^{an}})$

ON ALGEBRAIC SIDE, POINCARÉ LEMMA FAILS

EG $X = \mathrm{Spec}\, \mathbb{C}[t] = \mathbb{A}^1_{\mathbb{C}}$, $\omega = \dfrac{dt}{1+t}$ FORM NEAR ZERO HAS NO PRIMITIVE

$\rightsquigarrow \log(1+t)$ FORMALLY LIVES IN $\mathbb{C}[[t]]$

GROTHENDIECK'S IDEA: INTRODUCE INFINITESIMAL DEFORMATIONS.

§ INFINITESIMAL SITE

'MY PROPOSITIONS SERVE AS ELUCIDATIONS IN THE FOLLOWING WAY: ANYONE WHO UNDERSTANDS ME EVENTUALLY RECOGNIZES THEM AS NONSENSICAL, WHEN HE HAS USED THEM - AS STEPS - TO CLIMB BEYOND THEM.
(HE MUST, SO TO SPEAK, THROW AWAY THE LADDER AFTER HE HAS CLIMBED UP IT)
HE MUST TRASCEND THESE PROPOSITIONS, AND THEN HE WILL SEE THE WORLD ALRIGHT.'

(1921) L. WITTGENSTEIN, TRACTATUS LOGICO-PHILOSOPHICUS

LET $f: X \longrightarrow S$ MORPHISM OF SCHEMES (NO ASSUMPTION ON REGULARITY OR CHARACTERISTIC)

DEFINE $\mathrm{Inf}(X/S)$ INFINITESIMAL SITE

- OBJECTS: $(U \hookrightarrow T)$ W/
  - $U \subset X$ ZARISKI OPEN SUBSET
  - $U \hookrightarrow T$ NILPOTENT CLOSED IMMERSION S-SCHEMES

  I.E $\mathcal{I} = \ker(\mathcal{O}_T \longrightarrow \mathcal{O}_U)$, $\exists n > 0$ $\mathcal{I}^n = 0$.

- MORPHISMS : $(U,T) \longrightarrow (U',T')$ ARE COMMUTATIVE DIAGRAMS

$$\begin{tikzcd} U \arrow[r, "u"] \arrow[d] & U' \arrow[d] \\ T \arrow[r, "t"] & T' \end{tikzcd}$$

- $u: U \longrightarrow U'$ OPEN IMMERSION
- $t: T \longrightarrow T'$ S-MORPHISM

- COVERINGS : $\{(U_i, T_i) \longrightarrow (U,T)\}_i$ IS COVERING IF $t_i : T_i \longrightarrow T$ OPEN IMMERSION

$T = \bigcup T_i$ COVERING, $U_i = U \times_T T_i$.

PROP A SHEAF OF SETS ON $\mathrm{Inf}(X/S)$ CAN BE IDENTIFIED W/ SYSTEM OF ZARINSKI SHEAVES OF SETS $\{F_{(U,T)}\}_{(U,T)}$ ON T FOR EACH (U,T) IN $\mathrm{Inf}(X/S)$ TOGETHER W/, FOR EACH $(u,t): (U,T) \longrightarrow (U',T')$ A HOMOMORPHISM

$$\rho_t : t^{-1} F_{(U',T')} \longrightarrow F_{(U,T)}$$

SATISFYING

- TRANSITIVITY CONDITION FOR $(U,T) \longrightarrow (U',T') \longrightarrow (U'',T'')$
- ρ_t IS AN ISOMORPHISM FOR $t: T \longrightarrow T'$ OPEN IMMERSION.

PROOF EXERCISE. □

EG • STRUCTURE SHEAF $\mathcal{O}_{X_{\mathrm{inf}}}$

$$(U,T) \longmapsto \Gamma(T, \mathcal{O}_T)$$

'UN CRISTAL POSSÈDE DEUX PROPRIÉTÉS CARACTÉRISIQUES : LA RIGIDITÉ, ET LA FACULTÉ DE CROITRE, DANS UN VOISINAGE APPROPRIÉ. IL Y A DES CRISTAUX DE TOUTE ESPÈCE DE SUBSTANCE : DE CRISTAUX DE SOUDE, DE SOUFRE, DE MODULES, D'ANNEAUX, DE SCHÉMAS RELATIFS ETC'. MAY 1966, PISA

A. GROTHENDIECK - LETTER TO J. TATE

ASSUME X SMOOTH

~~PROP~~ def THE CATEGORY OF $\mathcal{O}_X$-mod ON X WITH INTEGRABLE CONNECTION RELATIVE TO S IS EQUIVALENT TO CATEGORY OF CRYSTALS OF $\mathcal{O}_{X_{\mathrm{inf}}}$-mod.

def A CRYSTAL OF $\mathcal{O}_{X_{\mathrm{inf}}}$-mod IS A SHEAF F OF $\mathcal{O}_{X_{\mathrm{inf}}}$ SUCH THAT FOR ANY MORPHISM $(U,T) \xrightarrow{(u,t)} (U',T')$ THE HOMOMORPHISM

$$\rho_t : t^{-1} F_{(U',T')} \longrightarrow F_{(U,T)}$$

IS AN ISOMORPHISM.

PROOF LET F AN $\mathcal{O}_X$-mod W/ INTEGRABLE CONNECTION

$$\nabla : F \longrightarrow F \otimes \Omega^1_{X/S}$$

LET (U,T) OBJECT IN $\mathrm{Inf}(X/S)$

$$\begin{tikzcd} X \arrow[r, hook] \arrow[d] & U \arrow[d] \\ S & T \arrow[ul, dashed, "g"'] \end{tikzcd}$$

BY SMOOTHNESS THERE EXIST RETRACTION $g : T \longrightarrow X$

DEFINE $\mathcal{F}_{(U,T)} = g^* F$.

IF $f_1, f_2 : T \longrightarrow X$ TWO DIFFERENT RETRACTION

$$(f_1, f_2) : T \longrightarrow X \times_S X$$

FACTORS THROUGH INFINITESIMAL NEIGHBORHOOD OF DIAGONAL

$$T \underset{\pi}{\longrightarrow} \Delta^n_{X/S} \longrightarrow X \times_S X$$

LET $p_i^{(n)} : \Delta_{X/S} \longrightarrow X$ TWO PROJECTIONS, THE DATA OF AN INTEGRABLE CONNECTION IS EQUIVALENT TO ISOMORPHISM

$$p_1^{(n)*}(F) \simeq p_2^{(n)*}(F)$$

$\rightsquigarrow f_1^*(F) \simeq f_2^*(F)$. REST OF PROOF EXERCISE. $\square$

REM TWO PATOLOGIES OF $\mathrm{Inf}(X/S)$

- IT HAS NO FINAL OBJECT (UNLIKE Top, Zar, Ét ecc)
- NO CANONICAL WAY TO PULLBACK THICKENINGS

$\rightsquigarrow$ MOVE TO <u>INFINITESIMAL TOPOS</u> $(X/S)_{\inf} := \mathrm{Sh}(\mathrm{Inf}(X/S))$

- FINAL OBJECT IN $(X/S)_{\inf}$ GIVEN BY $\{*\}$ CONSTAT PRESHEAF W/ SINGLETON

  $\rightsquigarrow$ NOT REPRESENTABLE

- PULLBACK OF THICKENINGS $f_{\inf} : (X'/S')_{\inf} \longrightarrow (X/S)_{\inf}$ PAIR OF ADJOINT FUNCTORS

$$\begin{tikzcd} X' \arrow[r, "f"] \arrow[d] & X \arrow[d] \\ S' \arrow[r] & S \end{tikzcd}$$

LET (U,T) AN $\mathrm{Inf}(X/S)$ SEEN AS SHEAF, DEFINE

$$f^{-1}(U,T) : (U',T') \mapsto \begin{cases} \emptyset & \text{IF } f(U') \not\subset U \\ \{h : T' \longrightarrow T \text{ MAKING } (*) \text{ COMMUTE}\} \end{cases}$$

$$\begin{tikzcd} U' \arrow[rr, "f"] \arrow[dr] \arrow[dd] & & U \arrow[dr] \arrow[dd] & \\ & T' \arrow[rr, dashed, "h"] \arrow[dl] & & T \arrow[dl] \\ S' \arrow[rr] & & S & \end{tikzcd}$$

$\widetilde{f^{-1}(U,T)}$ NOT REPRESENTABLE IN GENERAL

REM FOR $X \xrightarrow{id} X$, THE SHEAF $\tilde{X}^m$ CAN BE DESCRIBED AS LIMIT OF REPRESENTABLE SHEAVES

$$\tilde{X}^m = \varinjlim_i \widetilde{\Delta^i_{X/S}(m-1)},$$ $\Delta^i_{X/S}(j)$ i-TH INF. NEIGHBORHOOD OF DIAGONAL

$X \longrightarrow X \times_S X \times \cdots X$ j-TIMES

FACT $\tilde{X} \to \{*\}$ IS A COVERING OF THE FINAL OBJECT IN $(X/S)_{inf}$

⇝ FORM ČECH - ALEXANDER COMPLEX $\check{C}A^\bullet((\tilde{X} \to \{*\}), \mathcal{O}_{X_{inf}})$

$$\check{C}A^m((\tilde{X} \to \{*\}) = \varprojlim_i P^i_{X/S}(m)$$

W/ $P^i_{X/S}(m)$ STRUCTURE SHEAF OF $\Delta^i_{X/S}(m)$ i-TH INFINITESIMAL NEIGHBORHOOD OF DIAGONAL IN $X \times_S X \times_S \cdots X$ (m TIMES).

PROP X SMOOTH

$$H^\bullet_{inf}(X, \mathcal{O}_{X_{inf}}) \cong \mathbb{H}^\bullet(X_{Zar}, \check{C}A^m((\tilde{X} \to \{*\}), \mathcal{O}_{X_{inf}}))$$

THEOREM IF S IS $\mathbb{Q}$-SCHEME, X IS SMOOTH ON S, THERE IS A CANONICAL ISOMORPHISM

$$H^\bullet_{inf}(X, \mathcal{O}_{X_{inf}}) \simeq H^\bullet_{dR}(X/S).$$

REM THE PROOF OF THE THEOREM IS BASED ON THE INFINITESIMAL POINCARÉ LEMMA THAT SHOWS THAT SHEAVES IN ČECH - ALEXANDER COMPLEX CAN BE RESOLVED VIA A DE RHAM SEQUENCE (USING LINEARISING FUNCTORS)

NOTE EVERYTHING DONE BEFORE THE LAST THEOREM WORKS IN ARBITRARY CHARACTERISTIC

⇝ NEED TO FIND A WAY TO EXTEND POINCARÉ LEMMA TO p-ADIC SETTING

⇝ BERTHELOT'S IDEA (1974): INTRODUCE DIVIDED POWER STRUCTURES

SUGGESTED BY GROTHENDIECK, INSPIRED BY ROBY NORBERT '65

§ CRYSTALLINE SITE

def LET A RING, $I \subset A$ IDEAL. A DIVIDED POWER STRUCTURE ON I IS A COLLECTION OF MAPS $\gamma_n : I \to A$ FOR $n \geq 0$ ST

i) $\gamma_0(x) = 1, \quad \gamma_1(x) = x$

ii) $\gamma_n(x) \in I$ IF $n \geq 1$

iii) $\gamma_n(x+y) = \sum_{i+j=n} \gamma_i(x)\gamma_j(y)$

iv) $\gamma_n(\lambda x) = \lambda^n \gamma_n(x)$

v) $\gamma_n(x)\gamma_m(x) = \binom{m+n}{n} \gamma_{n+m}(x)$

vi) $\gamma_m(\gamma_n(x)) = \dfrac{(mn)!}{m!\,(n!)^m} \gamma_{mn}(x)$

REM FROM THE DEFINITION IT FOLLOWS $n!\,\gamma_n(x) = x^n$

⇝ THESE ARE EXACTLY THE ELEMENTS THAT ARE 'NEEDED' TO INTEGRATE

EG
- A $\mathbb{Q}$-ALGEBRA, EVERY IDEAL HAS UNIQUE PD STRUCTURE $\gamma_n(x) = x^n/n!$
- V DVR OF UNEQUAL CHARACTERISTIC, π UNIFORMIZER, e RAMIFICATION

  V HAS PD STRUCTURE IFF $e<p$
- A RING, UNIVERSAL PD-STRUCTURE IN r-VARIABLES $A\langle x_1,\dots,x_r\rangle$
  - GEN. AS A-mod BY SYMBOLS $x_1^{[i_1]}\cdots x_r^{[i_r]}$
  - PD STRUCTURE

    $$\gamma_n(x_i^{[1]}) = x_i^{[n]} \qquad \text{MORALLY} \quad x_i^{[n]} = x_i^n/n!$$
  - MULTIPLICATION $x_i^{[m]}x_i^{[n]} = \binom{m+n}{n} x_i^{[m+n]}$
  - GRADED RING STRUCTURE

    $\oplus\Gamma^n$, Γ^n GEN BY $x_1^{[i_1]}\cdots x_r^{[i_r]}$ W/ $i_1+\dots+i_r=n$

    $I = \oplus_{n\geq 1}\Gamma^n$

LEMMA (CRYSTALLINE POINCARÉ LEMMA)

LET A BE ANY RING, THE DE RHAM COMPLEX OVER $A[t_1,\dots,t_n]$ W/ VALUES IN $A\langle t_1,\dots,t_n\rangle$ IS A RESOLUTION OF A

PROOF $n=1$, THE COMPLEX $A\langle t\rangle \to A\langle t\rangle dt$ IS GIVEN BY

$$\sum_{k\geq 0} a_k t^{[k]} \mapsto \sum_{k\geq 1} a_k t^{[k-1]}dt \qquad d(\gamma_n(x)) = \gamma_{n-1}(x)dx$$

THEN WE HAVE EXACT SEQUENCE

$$0\to A\to A\langle t\rangle \to A\langle t\rangle dt \to 0.$$

SUPPOSE BY INDUCTION $A\to \omega^\bullet_{A\langle t_1,\dots,t_m\rangle}$ QUASI-ISOMORPHISM

TERMS OF $\omega^\bullet_{A\langle t_n\rangle}$ HAS LOCALLY FREE A-mod. THEN

$$A\otimes\omega^\bullet_{A\langle t_n\rangle} \to \omega^\bullet_{A\langle t_1,\dots,t_{n-1}\rangle}\otimes \omega^\bullet_{A\langle t_n\rangle} \quad \text{, QUASI-ISOMORPHISM}$$

$$A\to\omega^\bullet_{A\langle t_1,\dots,t_n\rangle} \quad \text{QUASI-ISOMORPHISM.}$$

□

WE CAN THEN TRY TO MODIFY INFINITESIMAL SITE TO INCORPORATE PD STRUCTURE

$\rightsquigarrow$ BERTHELOT '74 THESIS.

def LET k FIELD PERFECT OF char $p>0$, $W = W(k)$ WITT VECTORS RING

$W_n = W/p^n$, X SMOOTH k-SCHEME

DEFINE THE CRYSTALLINE SITE $Cris(X/W_n)$

- OBJECTS: COMMUTATIVE DIAGRAMS

$$\begin{tikzcd} U \arrow[r, "i"] \arrow[d] & V \arrow[d] \\ \operatorname{Spec} k \arrow[r] & \operatorname{Spec} W_n \end{tikzcd}$$

$U\subset X$ ZARISK OPEN, $i: U\hookrightarrow V$ PD-THICKENING

I.E CLOSED IMMERSION OF W_n-SCHEMES

W/ $\ker(\mathcal{O}_V\to\mathcal{O}_U)$ ENDOWED W/ PD-STRUCTURE δ

COMPATIBLE W/ CANONICAL PD-STRUCTURE $pW(k)$

- MORPHISMS : $(U,V,\delta) \longrightarrow (U',V',\delta')$ COMMUTATIVE DIAGRAMS W/

$U \hookrightarrow U'$ OPEN IMMERSION, $V \hookrightarrow V'$ MORPHISM COMP. W/ PD-STRUCTURES

- COVERINGS : AS FOR INFINITESIMAL SITE.

REM WE CAN DEFINE AND CHARACTERIZE SHEAVES AS IN INFINITESIMAL SITE

def CRYSTALLINE COHOMOLOGY IS DEFINED BY

$$H^i(X/W_n) = H^i((X/W_n)_{crys}, \mathcal{O}_{X/W_n}), \quad H^i(X/W) = \varprojlim H^i(X/W_n).$$

THEOREM IF Z/W_n IS A SMOOTH LIFT OF X, THE CRYSTALLINE COHOMOLOGY IS CANONICALLY ISOMORPHIC TO DE RHAM COHOMOLOGY

$$H^i(X/W_n) \simeq H^i_{dR}(Z/W_n)$$